# Weighted averages

1. Suppose the weight of the mid-term examination is 0.2, the weights of each of four projects is 0.1, and the weight of the final examination is 0.4. Suppose your mid-term grade is 63, your grades in your four projects are 85, 100, 92 and 88. What grade do you require on the final to get an 80 in the course?

Answer: 77¼

2. Recall that a convex combination of $n$ items is a weighted average where all weights are non-negative. Prove that all weights, therefore, cannot exceed 1.

Answer: Assume that the first weight is greater than 1, so $w_1 > 1$. If all other weights are non-negative, then $w_k \geq 0$, then $w_1 + w_2 + \cdots + w_n > 1 + 0 + \cdots + 0 > 1$, and thus, $w_1 + w_2 + \cdots + w_n \neq 1$, which contradicts our definition of a weighted average; that is, where the sum of the weights must be 1.

3. Can you take a weighted average of vectors?

Answer: Yes. The weights are scalars, and therefore a weighted average of vectors would be a linear combination of those vectors.

4. Suppose we had $n$ points $x_1, \ldots, x_n$. How would you describe all convex combinations of these $n$ points?

Answer: All points on the closed interval $[\min\{x_1, \ldots, x_n\}, \max\{x_1, \ldots, x_n\}]$

5. If a weighted average is not a convex combination, is it possible for a weighted average of two points $x_1$ and $x_2$ to fall outside the interval $[\min\{x_1, x_2\}, \max\{x_1, x_2\}]$?

Answer: Yes assuming that the two values are different. For example, if $x_1 = 0$ and $x_2 = 1$, and choose any real number $c$; then $w_1 = 1 - c$ and $w_2 = c$ defies a weighted average, as $w_1 + w_2 = (1 - c) + c = 1$, and $w_1x_1 + w_2x_2 = c$.

6. Suppose you had two vectors in $\mathbf{R}^2$, $\mathbf{u}_1$ and $\mathbf{u}_2$. How would you describe all possible weighted averages of these two vectors?

Answer: If these two vectors are equal, then all weighted averages equals this one vector. If the two vectors are unequal, then all weighted averages would equal the line that passes through these two vectors.

7. Suppose you had two different vectors in $\mathbf{R}^2$, $\mathbf{u}_1$ and $\mathbf{u}_2$. How would you describe all possible convex combination of these two vectors?

Answer: Because the two vectors are unequal, all convex combinations of these two vectors includes all points on the closed line segment that connects $\mathbf{u}_1$ to $\mathbf{u}_2$.

8. Suppose you had three vectors in $\mathbf{R}^2$, $\mathbf{u}_1$ , $\mathbf{u}_2$ and $\mathbf{u}_3$. How would you describe all possible weighted averages of these three vectors?

Answer: If these three vectors are equal, then all weighted averages equals this one vector. If the three vectors are not all equal but co-linear (all on the same line), then all weighted averages would equal that line on which these three vectors lie. If the three vectors are not even co-linear, than all weighted averages would equal all points in $\mathbf{R}^3$.

9. Suppose you had three vectors in $\mathbf{R}^2$, $\mathbf{u}_1$ , $\mathbf{u}_2$ and $\mathbf{u}_3$. How would you describe all possible convex combinations of these three vectors?

Answer: If these three vectors are equal, then all convex combinations equals this one vector. If the three vectors are not all equal but co-linear (all on the same line), then all weighted averages would equal that line segment that connects all three vectors. If the three vectors are not even co-linear, than all linear combinations would be the triangular region defined by connecting the three vectors.

Acknowledgement: Chinemerem Chigbo noted that Question 7 had ‘equal’ instead of ‘unequal’ in describing the two vectors.